High-Order Non-Oscillatory Compact Reconstruction Scheme for Overset Grids

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Motivation

Accurate numerical simulation of the wake flow field around a rotorcraft
- Long term convection and mutual interaction of vortices
- Interactions of vortices with fuselage and ground plane
- Accurate resolution of near-blade turbulent structures

High order accurate Navier-Stokes solver
- High spectral resolution for accurate capturing of smaller length scales
- Non-oscillatory solution across shock waves and shear layers
- Low dissipation errors for preservation of flow structures over large distances
The Compact-Reconstruction WENO (CRWENO) scheme

- Convex combination of \( r \)-th order candidate compact interpolations
- Optimal weights in smooth regions \( \to \) \((2r-1)\)-th order compact interpolation
- Smoothness - dependent weights \( \to \) Non-oscillatory interpolation for discontinuities

\[
f_{j+1/2} = \sum_{k=1}^{r} \omega_k f_j^{k+1/2}
\]

Why Compact Reconstruction?

- High order accuracy with smaller stencils
- Better spectral resolution than explicit interpolation (bandwidth resolving efficiency)
- Lower dissipation at resolved frequencies
- Taylor series error order of magnitude lower

Dispersion and dissipation relationships

$5^{th}$ Order CRWENO scheme

\[
\frac{2}{3} f_{j-1/2} + \frac{1}{3} f_{j+1/2} = \frac{1}{6} f_{j-1} + \frac{5}{6} f_{j}
\]

\[
\frac{1}{3} f_{j-1/2} + \frac{2}{3} f_{j+1/2} = \frac{5}{6} f_{j} + \frac{1}{6} f_{j+1}
\]

\[
\frac{2}{3} f_{j+1/2} + \frac{1}{3} f_{j+3/2} = \frac{1}{6} f_{j} + \frac{5}{6} f_{j+1}
\]

\[
\frac{3}{10} f_{j-1/2} + \frac{6}{10} f_{j+1/2} + \frac{1}{10} f_{j+3/2} = \frac{1}{30} f_{j-1} + \frac{19}{30} f_{j} + \frac{10}{30} f_{j+1}
\]

\[
\left(\frac{2}{3} \omega_1 + \frac{1}{3} \omega_2\right) f_{j-1/2} + \left(\frac{1}{3} \omega_1 + \frac{2}{3} (\omega_2 + \omega_3)\right) f_{j+1/2} + \frac{1}{3} \omega_3 f_{j+3/2} = \frac{\omega_1}{6} f_{j-1} + \frac{5(\omega_1 + \omega_2)}{6} f_{j} + \frac{\omega_2 + 5\omega_3}{6} f_{j+1}
\]
Baseline Solver

Integration of the CRWENO scheme with a compressible Navier Stokes solver for overset structured meshes

- **Time Marching:** 2\textsuperscript{nd} order Backward Differencing (BDF2) and 3\textsuperscript{rd} order Total Variation Diminishing Runge Kutta (TVDRK3)
- **Dual time-stepping** for time-accurate computations
- **Implicit Inversion:** Diagonalized ADI and LU-SGS
- **Spatial reconstruction:**
  - 5\textsuperscript{th} order CRWENO scheme (compact)
  - 3\textsuperscript{rd} order MUSCL and 5\textsuperscript{th} order WENO schemes (non-compact)
- **Upwinding:** Roe’s flux differencing
- **Turbulence Modeling:** Spallart-Almaras one-equation model
- **Implicit hole-cutting** for overset meshes
- **Viscous Terms** discretized by 2\textsuperscript{nd} order central differences
Applications

Scalar Conservation Laws

- Non-oscillatory solutions across discontinuities
- Absolute errors order of magnitude lower than WENO5 scheme
- Sharper resolution of extrema & shocks/contact discontinuities
- Significantly lower dissipation for smaller length scales
- Improved preservation of flow structures over large convection distances
- Validated for curvilinear meshes

Inviscid Euler Equations

- WENO5
- CRWENO5
Overset Grids

Solution algorithm on overset meshes
• Identification of field, overlap and hole regions
• Field points → Governing equations are solved
• Overlap region → Solution exchanged with other meshes
• Hole region → Blanked out, contains non-physical values
• Implicit Hole-Cutting (Lee & Baeder, 2008)
• Tri-linear interpolation of solution between donor and receiver points

Application of compact schemes
• Coupled solution for the interface fluxes
• Solution in hole region coupled with solution at field points
• System of equations contain non-physical values from the hole region
CRWENO on Overset Grids

Behavior across discontinuity ↔ Behavior across hole cut
(Non-physical values appear as a discontinuity)

\[
\begin{align*}
\frac{2}{3} f_{j-1/2} + \frac{1}{3} f_{j+1/2} &= \frac{1}{6} f_{j-1} + \frac{5}{6} f_j \\
\frac{1}{3} f_{j-1/2} + \frac{2}{3} f_{j+1/2} &= \frac{5}{6} f_j + \frac{1}{6} f_{j+1} \\
\frac{2}{3} f_{j+1/2} + \frac{1}{3} f_{j+3/2} &= \frac{1}{6} f_{j} + \frac{5}{6} f_{j+1}
\end{align*}
\]

\[\begin{bmatrix}
\omega_1 & \times & \times & 0 & 0 \\
\omega_2 & \times & 0 & 0 & \times \\
\omega_3 & 0 & 0 & \times & \times
\end{bmatrix}
\]

Decoupling of domain across a discontinuity

\[
\begin{bmatrix}
\omega_1 & \times & \times & 0 & 0 \\
\omega_2 & \times & 0 & 0 & \times \\
\omega_3 & 0 & 0 & \times & \times
\end{bmatrix}
= \begin{bmatrix}
\mathbf{f}_{1/2} \\
\mathbf{f}_{j+1/2} \\
\mathbf{f}_{j+3/2} \\
\mathbf{f}_{N+1/2}
\end{bmatrix}
\]

\[\mathbf{RHS}\]
CRWENO on Overset Grids (Contd.)

Adaptive stenciling of the CRWENO scheme

Decoupling of solution between field and hole points

Grid Line \((1: i_{\text{max}}, j)\)
Applications

• Verification / Validation
  – Isentropic vortex convection
  – Steady flow over SC2110 airfoil in wind tunnel, with and without leading edge slat
  – Dynamic stall of a pitching SC1095 airfoil in wind tunnel

• Application
  – Flow around the Harrington two-bladed rotor
Comparison of CRWENO5 and WENO5
Long Term Convection (1000 core radii)
Better preservation of vortex strength and shape by the CRWENO5 scheme

Comparison of pressure error at vortex core
Isentropic Vortex Convection on Overset Grids

Comparison of solutions on single and overset meshes for the CRWENO5 scheme (20 core radii) → Good agreement

Comparison of WENO5 and CRWENO5 on overset meshes (50 core radii)
SC2110 Airfoil w/ Slat in Wind Tunnel

### Wind Tunnel Mesh
- Clustered Cartesian, 151x101 points

### Airfoil Mesh
- C-type, 365x138 points

### Slat Mesh
- C-type, 317x97 points

### Flow Conditions:
- Reynolds number = 4.15e6
- Freestream Mach = 0.283
- Angle of Attack = 0°, 10°

### Verification of CRWENO5 scheme with non-compact MUSCL3 and WENO5 schemes

#### Pressure contours
(CRWENO5 w/ BDF2)

- α = 0°
- α = 10°
SC2110 Airfoil w/ Slat in Wind Tunnel

- High gradients in flow between slat and airfoil
- Overlap and solution transfer between slat and airfoil meshes
- Pressure contours from CRWENO5 compared with those from non-compact schemes
SC1095 Dynamic Stall

Flow Conditions:
Reynolds number: 3.92 million, Freestream Mach number 0.302
Mean angle of angle: 9.78°, Pitch Amplitude: 9.9°, Reduced Frequency: 0.099, Tunnel height: 5c

Numerical Solution: Time stepping: BDF2 w/ 15 sub-iterations

Airfoil Mesh – C-type, 365x138 points
Wind Tunnel Mesh – Clustered Cartesian, 151x101 points
Validation for Overset Meshes w/ Grid Motion

- **Pressure**
- **Vorticity Magnitude**

**MUSCL3**

**WENO5**

**CRWENO5**

Shed vortices from upper surface:
- Contour lines are continuous between airfoil and wind tunnel meshes
- Shed vortices pass smoothly between the two domains
Harrington 2-Bladed Rotor

Flow Conditions:
- $M_{\text{tip}}$: 0.352
- $Re_{\text{tip}}$: 3.5 million

Validation of thrust & power coefficients and figure of merit

Cylindrical Background Mesh
127 x 116 x 118 points

Rotor Geometry:
- Aspect Ratio – 8.33
- Airfoil section – NACA (t/c: 27.5% @ 0.2R, 15% @ R)
Harrington 2-Bladed Rotor
(Near-Blade and Wake Flowfield)

CRWENO5

MUSCL3

CRWENO5

MUSCL3
Conclusions and Future Work

CRWENO5 scheme validated and verified for overset grids

- Improved resolution of flow features due to lower numerical errors
- Slight loss of accuracy due to 2\textsuperscript{nd} order interpolation between meshes
- Non-physical solution in “hole” does not pollute field solution
- Smooth transfer of solution between different grids

Future Work & Applications of CRWENO5

- Application to meshes w/ immersed boundaries
- Wake flow from coaxial configurations
- Rotorcraft wake flow when operating “in-ground-effect” (IGE)
- Accurate modeling of wake vortex interactions with ground plane
- Application of CRWENO5 scheme with Vortex-Tracking Grids (VTGs)
- Sound generation due to blade vortex interaction (BVI) for rotor in forward flight
Thank You!
Questions?