A Multifluid Numerical Algorithm for Interpenetrating Plasma Dynamics

Debojyoti Ghosh[1] Christos Kavouklis[2] Richard L. Berger[3]
Thomas Chapman[3] Jeffrey A. F. Hittinger[1]

[1] *Center for Applied Scientific Computing* [2] *Computational Engineering* [3] *WCI*

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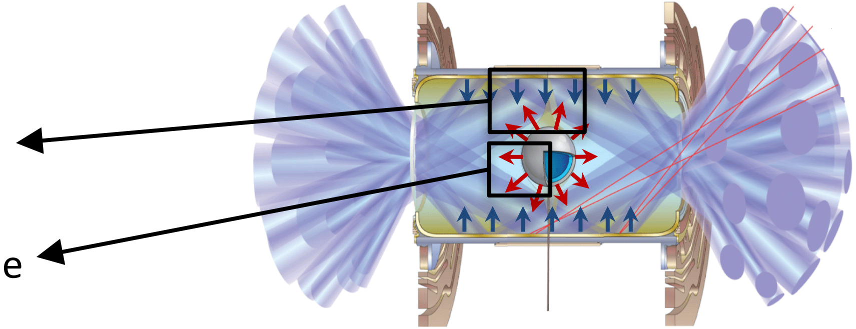
Motivation

Inertial Confinement Fusion: **Colliding plasmas from hohlraum wall and capsule**

Interpenetration of plasma flows from capsule and hohlraum wall

- Large range of Z: $2 \leq Z \leq 60$
- Supersonic flows ($\Delta u \approx 10^8$ cm/s)

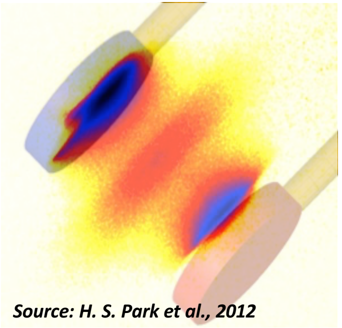
Species separation inside target capsule



Source: https://csdl-images.computer.org/mags/cs/2014/06/figures/mcs20140600421.gif

Basic Physics Experiments

HED collisionless shock experiments



Source: H. S. Park et al., 2012

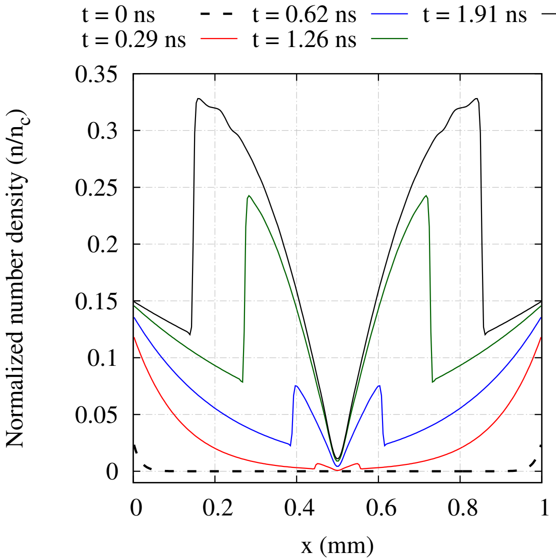
Multifluid phenomena that we want to model

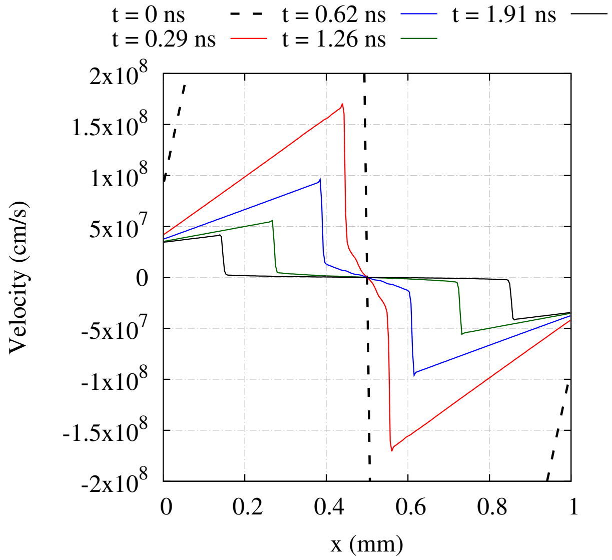
Interpenetrating plasmas

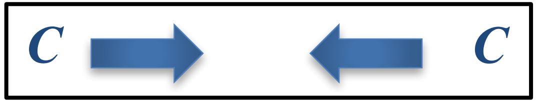
Plasma species separation

Why Not Single Fluid Model?

Single-fluid simulation of *colliding carbon plasma streams*







Initial and boundary conditions:
Expansion fan inflows at x = 0, 1;
Vacuum inside the domain

Unphysical solution

- Lack of distinct velocity fields for each species
- **Stagnation and density pile-up, shocks**
- Push-back of incoming plasma stream

Single-fluid models lack key physics to model plasma interpenetration

Multifluid Model

Inviscid Euler equations for each species

$$\alpha = 1, \cdots, n_s$$

$$\frac{\partial \rho_\alpha}{\partial t} + \nabla \cdot (\rho_\alpha \mathbf{u}_\alpha) = 0$$

$$\frac{\partial \rho_\alpha \mathbf{u}_\alpha}{\partial t} + \nabla (P_\alpha + \rho_\alpha \mathbf{u}_\alpha \otimes \mathbf{u}_\alpha) = -Z_\alpha e n_\alpha \nabla \phi + \sum_{\beta \neq \alpha} \mathbf{R}_{\alpha,\beta}$$

$$\frac{\partial \mathcal{E}_\alpha}{\partial t} + \nabla \cdot [(\mathcal{E}_\alpha + P_\alpha) \mathbf{u}_\alpha] = -Z_\alpha e n_\alpha \mathbf{u}_\alpha \cdot \nabla \phi + \sum_{\beta \neq \alpha} (\mathbf{R}_{\alpha,\beta} \cdot \mathbf{u}_\alpha + Q_{\alpha,\beta})$$

Interaction between species

- ***Distinct flows*** for each ion species and electrons
- All-species coupling via *friction* (collisions and kinetic processes) and *electric fields*
- Ion species can ***separate*** if friction weak & charge/mass ratios differ

Frictional drag

$$\mathbf{R}_{\alpha,\beta} = m_\alpha n_\alpha \nu_{\alpha,\beta} (\mathbf{u}_\beta - \mathbf{u}_\alpha)$$

Frictional heating and thermal equilibration

$$Q_{\alpha,\beta} = Q^{\mathrm{fric}}_{\alpha,\beta} + Q^{\mathrm{eq}}_{\alpha,\beta}$$

$$Q^{\mathrm{fric}}_{\alpha,\beta} = m_{\alpha,\beta} n_\alpha \nu_{\alpha,\beta} (\mathbf{u}_\beta - \mathbf{u}_\alpha)^2$$

$$Q^{\mathrm{eq}}_{\alpha,\beta} = -3 m_\alpha n_\alpha \frac{\nu_{\alpha,\beta}}{m_\alpha + m_\beta} (T_\alpha - T_\beta)$$

Fluid Electron Model

Solve the **Euler equations for the electrons** along with the ion species

$$\frac{\partial \rho_e}{\partial t} + \nabla \cdot (\rho_e \mathbf{v}_e) = 0$$

$$\frac{\partial \rho_e \mathbf{v}_e}{\partial t} + \nabla \left(P_e + \rho_e \mathbf{v}_e \otimes \mathbf{v}_e \right) = e \frac{\rho_e}{m_e} \nabla \phi - \sum_\alpha \mathbf{R}_{\alpha,e}$$

$$\frac{\partial \mathcal{E}_e}{\partial t} + \nabla \cdot \left[\left(\mathcal{E}_e + P_e \right) \mathbf{v}_e \right] = \sum_\alpha Q_{\alpha,e} - \nabla \cdot \mathbf{q}_e$$

Electron-ion friction

Electron-ion friction heating and thermal equilibration

Poisson equation for electrostatic potential

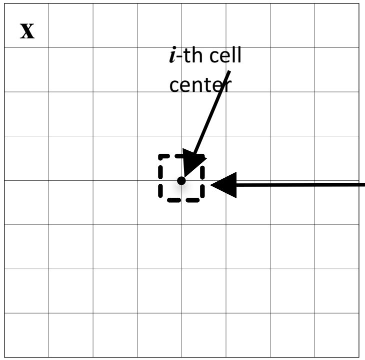
$$\nabla^2 \phi = 4\pi e \left(n_e - \sum_\alpha Z_\alpha n_\alpha \right)$$

Numerical concerns:

- Solution of a Poisson equation required for every RHS evaluation
- Electron thermal velocities much higher than flow velocities → **Stiff time scales** require implicit time integration

Numerical Method

4th order finite-volume discretization (using the *CHOMBO* library)



Domain $\Omega \equiv \{\mathbf{x} : 0 \leq \mathbf{x} \cdot \mathbf{e}_d \leq L_d, 1 \leq d \leq 3\}$

discretized into computational cells

$$\omega_{\mathbf{i}} = \prod_{d=1}^{3} \left[\left(\mathbf{i} - \frac{1}{2}\mathbf{e}_d \right) h, \left(\mathbf{i} + \frac{1}{2}\mathbf{e}_d \right) h \right]$$

$\mathbf{i}$: 3-dimensional integer index *(i, j, k)*
h: grid spacing

$$\mathbf{u} = \begin{bmatrix} \vdots \\ \rho_\alpha \\ \rho_\alpha \mathbf{v}_\alpha \\ \mathcal{E}_\alpha \\ \vdots \end{bmatrix}$$

Integral form of the governing equations

$$\frac{\partial}{\partial t} \left(\int_{\mathbf{X}(\omega_{\mathbf{i}})} \mathbf{u} d\mathbf{x} \right) = \int_{\partial \mathbf{X}(\omega_{\mathbf{i}})} \mathbf{F}(\mathbf{u}) \, d\mathbf{x}$$

Spatially-discretized ODE in time (integrated in time using **4th order Runge-Kutta method**)

$$\frac{\partial \bar{\mathbf{u}}_{\mathbf{i}}}{\partial t} = \frac{1}{h} \sum_{d=1}^{3} \left(\left\langle \hat{\mathbf{F}}_{\mathbf{i}+\frac{1}{2}\mathbf{e}_d} \right\rangle - \left\langle \hat{\mathbf{F}}_{\mathbf{i}-\frac{1}{2}\mathbf{e}_d} \right\rangle \right)$$

Cell-averaged solution

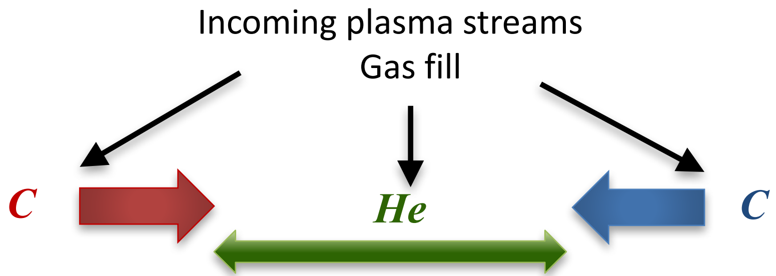
Face-averaged fluxes (computed using 4th order spatial discretization)

Preliminary Results

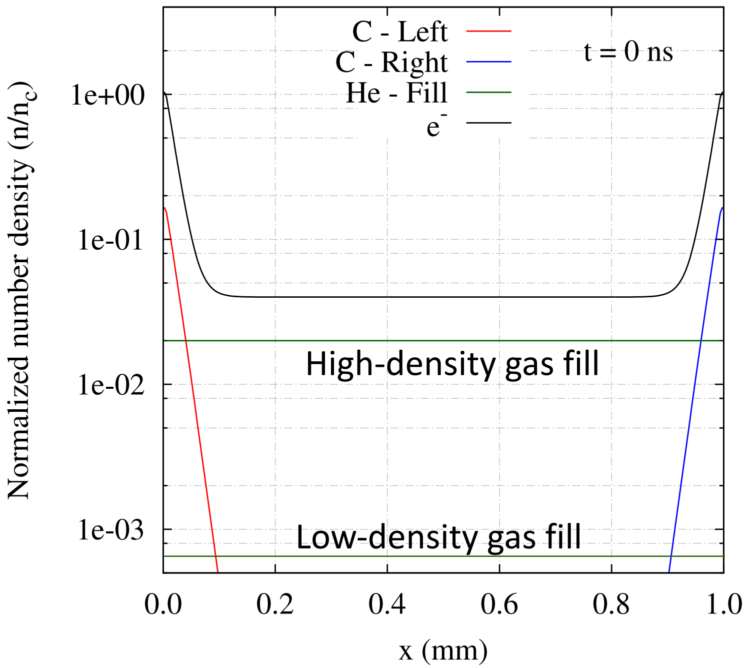
3D multifluid code currently **under development**

- 1D two-fluid code (*M. Khodak et al., APS DPP Annual Meeting, 2015*) extended to n fluids
- **Reduced electron model:** quasi-neutral plasma, inertia-less and isothermal electrons

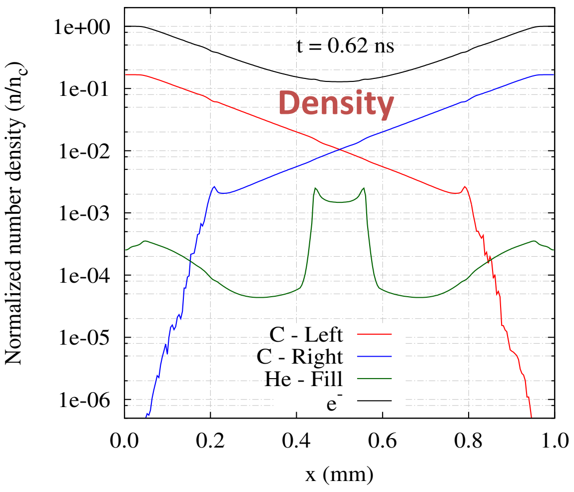
Simulate two interpenetrating plasmas in the presence of a gas fill

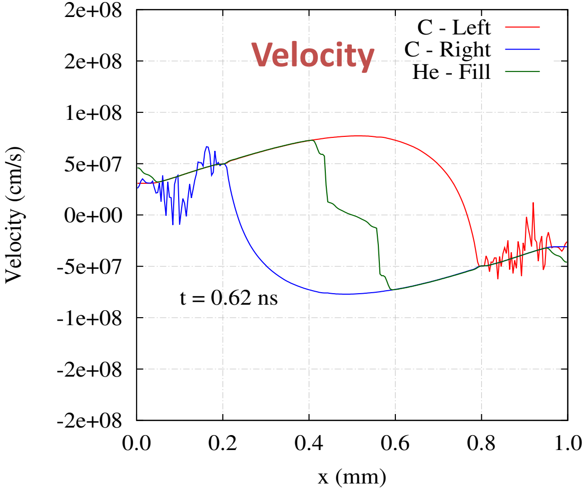


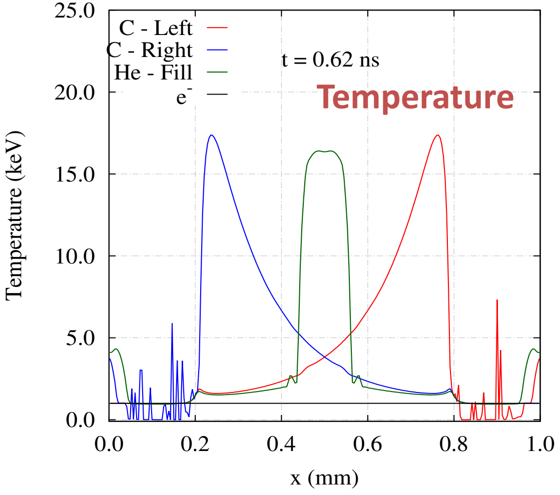
Initial Solution (Density)



Preliminary Results (Low Density Gas Fill)

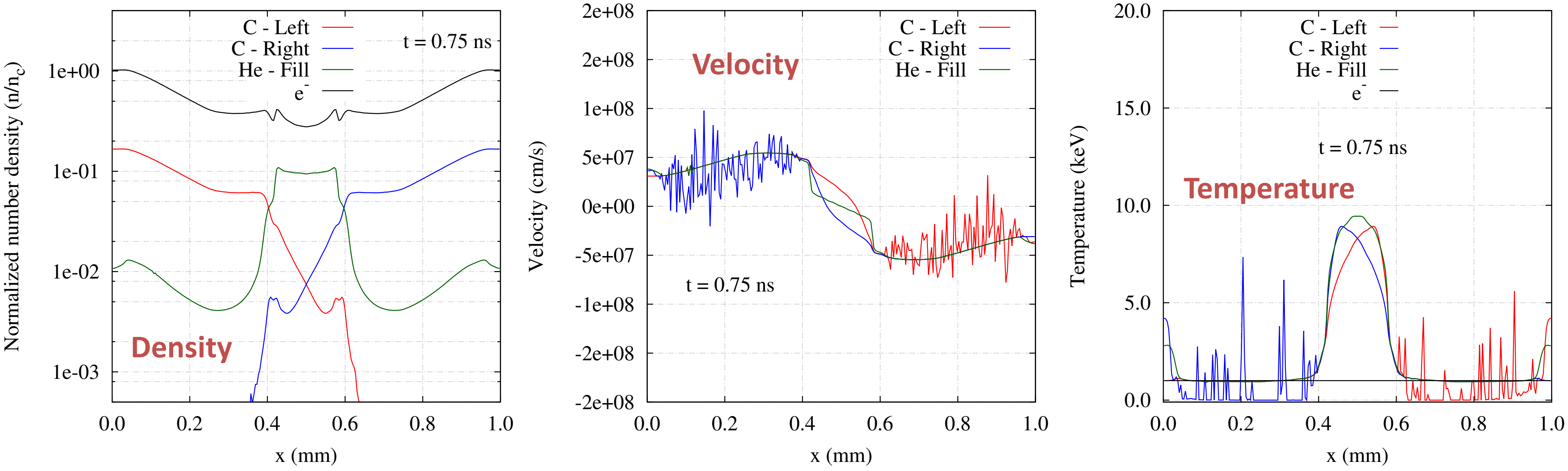






- Multifluid model *allows streams to interpenetrate*
- Helium density is insufficient to resist the carbon streams; streams *have not converged to a single velocity and temperature*
- Carbon-carbon frictional drag is the dominant collisional process

Preliminary Results (High Density Gas Fill)



- Higher collisionality results in *higher drag between the species*
- All three flows quickly *converge to a single flow velocity and temperature* (approaching **single fluid limit**)

Conclusions

- **Multifluid approach is able to capture the interpenetration effects**
  - ✓ Preliminary results from 1D code show *interpenetration is an important effect in ICF conditions*
  - ✓ Experimental scale interpenetration simulation (3D) **computationally feasible** with using a multifluid model

- **Future work**
  - Continue development of 3D code and validate it with experimental results
  - Explore *implicit or semi-implicit time integration methods* to treat stiff terms resulting from fluid electron model and heat flow terms
  - Incorporate *reduced kinetic model of ion-acoustic wave drag* (see poster in session PP11: *Joseph et al., Multiscale Models for the Two-Stream Instability*)

Thank you.
Questions?

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Reduced Electron Model

Isothermal electrons

$$P_e = n_e T_e; \quad T_e = \text{constant}$$

Quasi-neutrality

$$n_e = \sum_{\alpha=1}^{n_s} Z_\alpha n_\alpha,$$

$$\mathbf{v}_e = \frac{1}{n_e} \sum_{\alpha=1}^{n_s} Z_\alpha n_\alpha \mathbf{v}_\alpha$$

Inertia-less electrons

$$\nabla P_e = e n_e \nabla \phi + \sum_{\alpha=1}^{n_s} R_{\alpha,e}$$